Paper Reference(s)

6679

Edexcel GCE

Mechanics M3

Advanced Level

Friday 29 January 2010 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink or Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M3), the paper reference (6679), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

M35105A

1. A particle P of mass 0.5 kg is moving along the positive x-axis. At time t seconds, P is moving under the action of a single force of magnitude $[4 + (\cos \pi t)]$ N, directed away from the origin. When $t = 1$, the particle P is moving away from the origin with speed 6 m s^{-1}.

Find the speed of P when $t = 1.5$, giving your answer to 3 significant figures.

(7)

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2. A particle P moves in a straight line with simple harmonic motion of period 2.4 s about a fixed origin O. At time t seconds the speed of P is v m s^{-1}. When $t = 0$, P is at O. When $t = 0.4$, $v = 4$. Find

(*a*) the greatest speed of P,

(7)

(*b*) the magnitude of the greatest acceleration of P.

(2)

3.

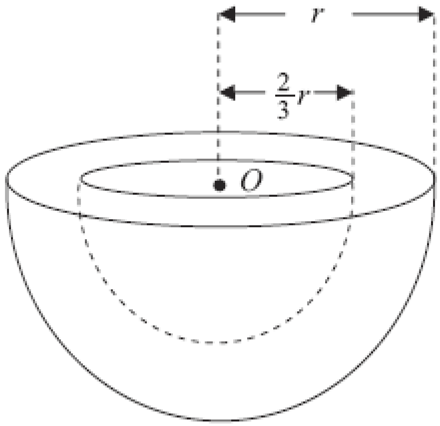

Figure 1

A bowl B consists of a uniform solid hemisphere, of radius r and centre O, from which is removed a solid hemisphere, of radius $\frac{2}{3}r$ and centre O, as shown in Figure 1.

(*a*) Show that the distance of the centre of mass of B from O is $\frac{65}{152}r$.

(5)

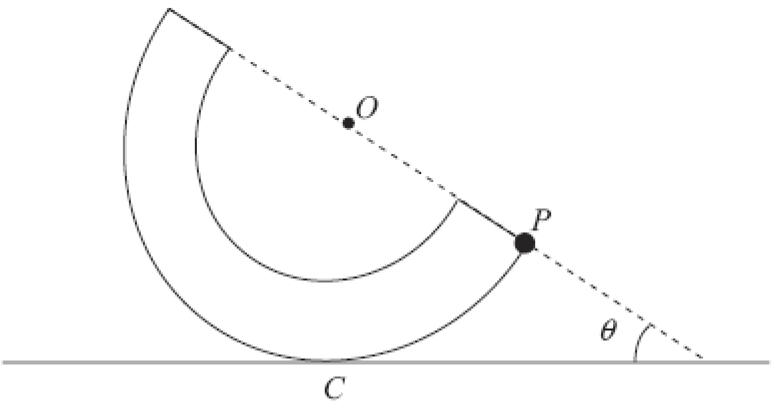

Figure 2

The bowl B has mass M. A particle of mass kM is attached to a point P on the outer rim of B. The system is placed with a point C on its outer curved surface in contact with a horizontal plane. The system is in equilibrium with P, O and C in the same vertical plane. The line OP makes an angle θ with the horizontal as shown in Figure 2. Given that $\tan\theta = \frac{4}{5}$,

(*b*) find the exact value of k.

(5)

4.

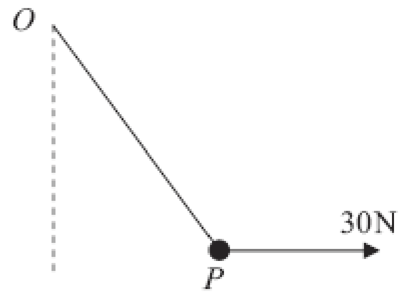

Figure 3

A particle P of weight 40 N is attached to one end of a light elastic string of natural length 0.5 m. The other end of the string is attached to a fixed point O. A horizontal force of magnitude 30 N is applied to P, as shown in Figure 3. The particle P is in equilibrium and the elastic energy stored in the string is 10 J.

Calculate the length OP.

(10)

5.

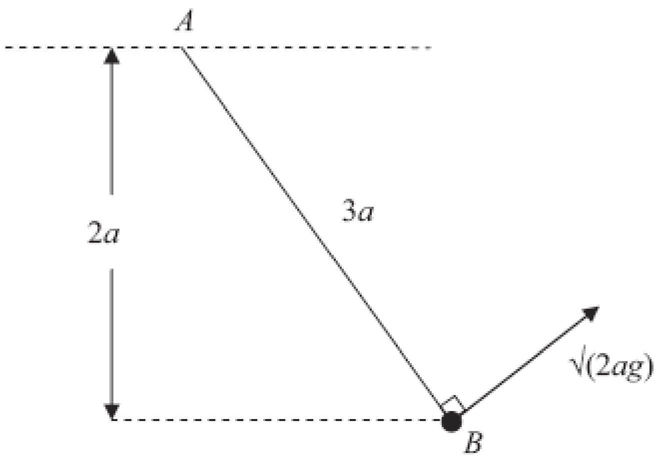

Figure 4

One end A of a light inextensible string of length $3a$ is attached to a fixed point. A particle of mass m is attached to the other end B of the string. The particle is held in equilibrium at a distance $2a$ below the horizontal through A, with the string taut. The particle is then projected with speed $\sqrt{(2ag)}$, in the direction perpendicular to AB, in the vertical plane containing A and B, as shown in Figure 4. In the subsequent motion the string remains taut. When AB is at an angle θ below the horizontal, the speed of the particle is v and the tension in the string is T.

(a) Show that $v^2 = 2ag(3 \sin \theta - 1)$.

(5)

(b) Find the range of values of T.

(6)

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6. A bend of a race track is modelled as an arc of a horizontal circle of radius 120 m. The track is not banked at the bend. The maximum speed at which a motorcycle can be ridden round the bend without slipping sideways is 28 m s^{-1}. The motorcycle and its rider are modelled as a particle and air resistance is assumed to be negligible.

(*a*) Show that the coefficient of friction between the motorcycle and the track is $\frac{2}{3}$.

(6)

The bend is now reconstructed so that the track is banked at an angle α to the horizontal. The maximum speed at which the motorcycle can now be ridden round the bend without slipping sideways is 35 m s^{-1}. The radius of the bend and the coefficient of friction between the motorcycle and the track are unchanged.

(*b*) Find the value of $\tan \alpha$.

(8)

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7. A light elastic string has natural length a and modulus of elasticity $\frac{3}{2}mg$. A particle P of mass m is attached to one end of the string. The other end of the string is attached to a fixed point A. The particle is released from rest at A and falls vertically. When P has fallen a distance $a + x$, where $x > 0$, the speed of P is v.

(*a*) Show that

$$v^2 = 2g(a + x) - \frac{3gx^2}{2a}.$$

(4)

(*b*) Find the greatest speed attained by P as it falls.

(4)

After release, P next comes to instantaneous rest at a point D.

(*c*) Find the magnitude of the acceleration of P at D.

(6)

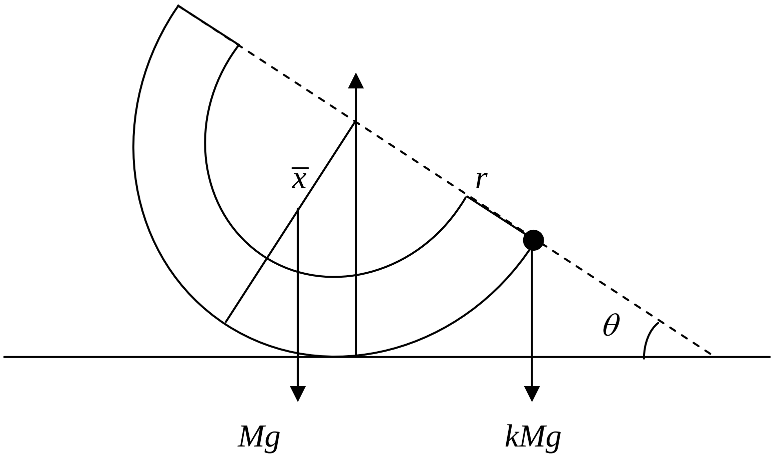
TOTAL FOR PAPER: 75 MARKS

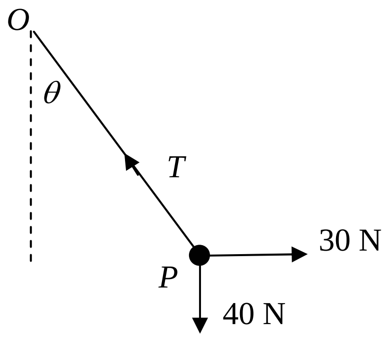
END

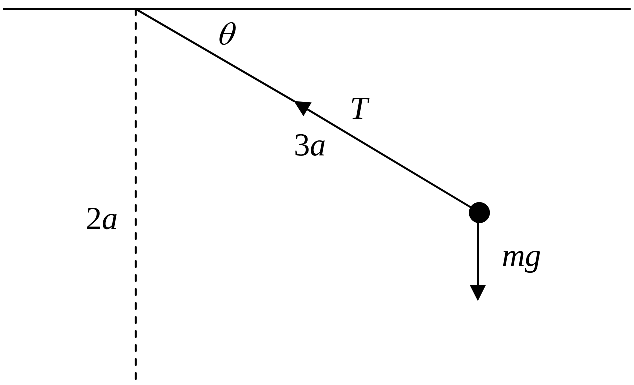
January 2010
6679 Mechanics M3
Mark Scheme

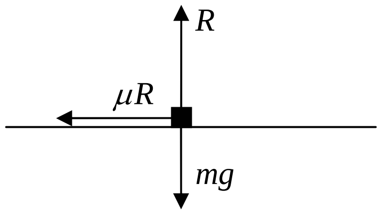
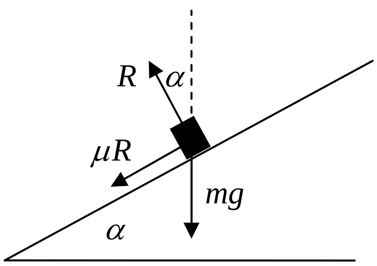
Question Number	Scheme	Marks
Q1.	$0.5a = 4 + \cos(\pi t)$	B1
	Integrating $\quad 0.5v = 4t + \dfrac{\sin(\pi t)}{\pi} \quad (+C)$	M1 A1
	Using boundary values $3 = 4 + C \Rightarrow C = -1$	M1 A1
	When $t = 1.5$ $0.5v = 6 - \dfrac{1}{\pi} - 1$	M1
	$v \approx 9.36 \ (\text{m s}^{-1})$ cao	A1 (7)
		[7]

Question Number	Scheme	Marks	
Q2.	(a) $\frac{2\pi}{\omega} = 2.4 \Rightarrow \omega = \frac{5\pi}{6} \left(\approx 2.62\right)$	M1 A1	
	$x = 0, t = 0 \Rightarrow x = a\sin\omega t$		
	when $t = 0.4$, $x = a\sin\left(\frac{5\pi}{6}\times 0.4\right) \quad \left(= \frac{\sqrt{3}}{2}a\right)$	M1	
	$v^2 = \omega^2\left(a^2 - x^2\right) \Rightarrow 16 = \frac{25\pi^2}{36}\left(a^2 - \frac{3a^2}{4}\right) \Rightarrow a = \frac{48}{5\pi} \left(\approx 3.06\right)$	M1 A1	
	$v_{\max} = a\omega = 8$ (or awrt 8.0 if decimals used earlier) cao	M1 A1	(7)
	(b) $\ddot{x}_{\max} = a\omega^2 = \frac{20\pi}{3}$ awrt 21	M1 A1	(2)
			[9]
	Alternative in (a)		
	(a) $\frac{2\pi}{\omega} = 2.4 \Rightarrow \omega = \frac{5\pi}{6}$	M1 A1	
	$x = 0, t = 0 \Rightarrow x = a\sin\omega t$		
	$\dot{x} = a\omega\cos\omega t$	M1	
	$4 = a\omega\cos\left(\frac{5\pi}{6}\times 0.4\right)$	M1	
	$a = \frac{48}{5\pi} \left(\approx 3.06\right)$ or $a\omega = 8$	A1	
	$v_{\max} = a\omega = 8$	M1 A1	(7)

Question Number	Scheme	Marks
Q3.	(a) Mass ratios: s 8, B 19, S 27 — anything in correct ratio	B1
	$\bar{x}$: $\frac{3}{8}\times\frac{2}{3}r$, $\bar{x}$, $\frac{3}{8}r$	B1
	$8\times\frac{1}{4}r+19\bar{x}=27\times\frac{3}{8}r$	M1 A1ft
	$\bar{x}=\frac{65}{152}r$ *	A1 (5)
	(b) 	
	$Mg\times\bar{x}\sin\theta=kMg\times r\cos\theta$	M1 A1=A1
	leading to $k=\frac{13}{38}$	M1 A1 (5)
		[10]

Question Number	Scheme	Marks	
Q4.			
	↑ $T\cos\theta = 40$ M1 attempt at both equations	M1 A1	
	→ $T\sin\theta = 30$	A1	
	leading to $T = 50$	M1 A1	
	$E = \dfrac{\lambda x^2}{2a} = 10$	B1	
	HL $T = \dfrac{\lambda x}{a} = 50$	M1	
	leading to $x = 0.4$	M1 A1	
	$OP = 0.5 + 0.4 = 0.9\,(\text{m})$	A1ft	(10)
			[10]

Question Number	Scheme	Marks
Q5.	(a) 	
	$\frac{1}{2}m\times 2ag-\frac{1}{2}mv^2=mg\left(2a-3a\sin\theta\right)$	M1 A1=A1
	leading to $v^2=2ga\left(3\sin\theta-1\right)$ * cso	M1 A1 (5)
	(b) minimum value of T is when $v = 0$ $\Rightarrow$ $\sin\theta=\frac{1}{3}$	B1
	$T=mg\sin\theta=\frac{mg}{3}$	M1 A1
	maximum value of T is when $\theta=\frac{\pi}{2}$ $\left(v^2=4ag\right)$	
	$\uparrow$ $T=\frac{mv^2}{3a}+mg$	M1 A1
	$=\frac{7mg}{3}$	A1 (6)
	$\left(\frac{mg}{3}\le T\le\frac{7mg}{3}\right)$	**[11]**

Question Number	Scheme	Marks
Q6.	(a) 	
	↑ $R = mg$	B1
	Use of limiting friction, $F_r = \mu R$	B1
	← $\mu R = \frac{m28^2}{120}$	M1 A1
	$\mu = \frac{28^2}{120 \times 9.8} = \frac{2}{3}$ ✱ cao	M1 A1 (6)
	(b) 	
	↑ $R\cos\alpha - \mu R\sin\alpha = mg$	M1 A1
	← $\mu R\cos\alpha + R\sin\alpha = \frac{mv^2}{r}$	M1 A1
	$\frac{\mu\cos\alpha + \sin\alpha}{\cos\alpha - \mu\sin\alpha} = \frac{v^2}{rg}$ Eliminating R	M1
	$\frac{2\cos\alpha + 3\sin\alpha}{3\cos\alpha - 2\sin\alpha} = \frac{25}{24}$ Substituting values	M1
	leading to $\tan\alpha = \frac{27}{122}$ awrt 0.22	M1 A1 (8)
		[14]

Question Number	Scheme	Marks
Q7.	(a)	
	$\frac{1}{2}mv^2+\frac{3mgx^2}{4a}=mg(a+x)$	M1 A2 (1, 0)
	leading to $v^2=2g(a+x)-\frac{3gx^2}{2a}$ * cso	A1 (4)
	(b) Greatest speed is when the acceleration is zero	
	$T=\frac{\lambda x}{a}=\frac{3mgx}{2a}=mg \Rightarrow x=\frac{2a}{3}$	M1 A1
	$v^2=2g\left(a+\frac{2a}{3}\right)-\frac{3g}{2a}\times\left(\frac{2a}{3}\right)^2 \left(=\frac{8ag}{3}\right)$	M1
	$v=\frac{2}{3}\sqrt{(6ag)}$ accept exact equivalents	A1 (4)
	(c) $v=0 \Rightarrow 2g(a+x)-\frac{3gx^2}{2a}=0$	M1
	$3x^2-4ax-4a^2=(x-2a)(3x+2a)=0$	
	$x=2a$	M1 A1
	At D, $m\ddot{x}=mg-\frac{\lambda\times 2a}{a}$ ft their $2a$	M1 A1ft
	$\lvert\ddot{x}\rvert=2g$	A1 (6)
		[14]
	Alternative to (b)	
	$v^2=2g(a+x)-\frac{3gx^2}{2a}$	
	Differentiating with respect to x	
	$2v\frac{dv}{dx}=2g-\frac{3gx}{a}$	
	$\frac{dv}{dx}=0 \Rightarrow x=\frac{2a}{3}$	M1 A1
	$v^2=2g\left(a+\frac{2a}{3}\right)-\frac{3g}{2a}\times\left(\frac{2a}{3}\right)^2 \left(=\frac{8ag}{3}\right)$	M1
	$v=\frac{2}{3}\sqrt{(6ag)}$ accept exact equivalents	A1 (4)

Question Number	Scheme	Marks
Q7.	*Alternative approach using SHM for* (b) *and* (c) If SHM is used mark (b) and (c) together placing the marks in the gird as shown.	
	Establishment of equilibrium position $T = \frac{\lambda x}{a} = \frac{3mge}{2a} = mg \quad \Rightarrow \quad e = \frac{2a}{3}$	bM1 bA1
	N2L , using y for displacement from equilibrium position $m\ddot{y} = mg - \frac{\frac{3}{2}mg\left(y+e\right)}{a} = -\frac{3g}{2a}y$ $\omega^2 = \frac{3g}{2a}$	bM1 bA1
	Speed at end of free fall $\quad u^2 = 2ga$	cM1
	Using A for amplitude and $v^2 = \omega^2\left(a^2 - x^2\right)$ $u^2 = 2ga$ when $y = -\frac{2}{3}a \quad \Rightarrow \quad 2ga = \frac{3g}{2a}\left(A^2 - \frac{4a^2}{9}\right)$	cM1
	$A = \frac{4a}{3}$	cA1
	Maximum speed $\quad A\omega = \frac{4a}{3} \times \sqrt{\left(\frac{3g}{2a}\right)} = \frac{2}{3}\sqrt{\left(6ag\right)}$	cM1 cA1
	Maximum acceleration $\quad A\omega^2 = \frac{4a}{3} \times \frac{3g}{2a} = 2g$	cA1